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Different Ways of Looking at the Electromagnetic Vacuum*

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Abstract

Some thoughts on the electromagnetic vacuum are presented in connection with the vacuum and source fields as alternative physical bases for understanding spontaneous emission, the Lamb shift, Casimir effects, van der Waals forces, and the "thermalization" of vacuum fluctuations for a uniformly accelerated observer.

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> And sometimes it seemed that something never yet seen yet long desired was about to happen, that a veil would drop from it all; but then it passed, nothing happened, the riddle remained unsolved, the secret spell unbroken...and still one knew nothing perhaps, was still waiting and listening.
> - Hermann Hesse, Narcissus and Goldmund

1. The Purpose of this Essay

Field theory has taught us that vacuum is not a tranquil state of nothingness, but rather a quantum state with fluctuations and physical consequences. Alas, the universe itself may have sprung from a quantum fluctuation. I shall discuss the electromagnetic vacuum. The discussion is arranged according to the following outline:

2. Einstein and the Vacuum
3. Spontaneous Emission
4. The Lamb Shift
5. Casimir Effects
6. Van der Waals Forces
7. Cavity-Modified Spontaneous Emission
8. Two Sides of a Coin
9. Commutators
10. Accelerated Observers
11. Where Do We Stand?

Among the questions I address are the following: What evidence do we have that

vacuum fluctuations have real physical consequences? Are there different ways of thinking about these effects? Is there more to be learned from the electromagnetic vacuum?

2. Einstein and the Vacuum

It would be interesting if a historian would trace the development of our present concept of the vacuum. It is not hard to find little examples of how vacuum fluctuations came close to being uncovered before the advent of quantum theory. For instance, Mulliken in 1924 found that by including zero-point energy he could get a better fit to molecular vibrational spectra. As another example I will consider briefly a paper published by Einstein and Stern [1] in 1913, around the middle of Einstein's long struggle to understand the Planck spectrum.

The model considered by Einstein and Stern is that of Einstein and Hopf, [2] and consists simply of dipole oscillators free to move in one dimension. The interaction with radiation increases the kinetic energy of the dipoles, but there is also a velocity-dependent force acting to slow down a particle moving through the field. In equilibrium the average rates at which these two effects change the kinetic energy must cancel, and this condition yields an equation for the (thermal) equilibrium spectral energy density $\rho(\omega)$. The equation is

$$\rho(\omega) - \frac{\omega}{3}\frac{d\rho}{d\omega} = \frac{1}{3kT}\rho(\omega) \langle H \rangle \qquad (2.1)$$

where $\langle H \rangle$ is the average kinetic energy of a dipole oscillator of frequency ω in equilibrium, and may be related to $\rho(\omega)$ by the equation

$$\langle H \rangle = \pi^2 c^3 \rho(\omega)/\omega^2 \qquad (2.2)$$

The solution of (2.1) is then the Rayleigh-Jeans spectrum:

$$\rho(\omega) = \omega^2 kT/\pi^2 c^3 \qquad (2.3)$$

which appears in this way to be an inevitable consequence of classical physics. However, Einstein and Stern remarked that if a dipole oscillator somehow has a zero-point (T = 0) energy $\hbar\omega$, so that $\langle H \rangle$ is replaced by (2.2) plus $\hbar\omega$ in (2.1), then the equation for $\rho(\omega)$ becomes

$$\rho(\omega) - \frac{\omega}{3}\frac{d\rho}{d\omega} = \pi^2 c^3 \rho(\omega)^2/3\omega^2 kT + \hbar\omega\rho(\omega)/3kT \qquad (2.4)$$

and the solution of this equation is the Planck formula.

Now it is not difficult to treat the Einstein-Hopf model fully quantum mechanically, and so to understand why the Einstein-Stern ansatz works: in quantum theory both a dipole oscillator and a field mode of frequency ω have a zero-point energy $\frac{1}{2}\hbar\omega$, and it turns out that the sum, $\frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega = \hbar\omega$, is just what Einstein and Stern found would have to be added to the classical theory to get the Planck formula. [3] Thus it appears that the zero-point energy of a field mode (and any other harmonic oscillator) must be $\frac{1}{2}\hbar\omega$ in order for the Planck formula to hold. Or, looked at another way, the validity of the Planck formula implies the existence of zero-point ("vacuum") field energy.

3. Spontaneous Emission

Since in free space there are $\omega^2 d\omega/\pi^2 c^3$ modes of the field per unit volume in the frequency interval $[\omega, \omega + d\omega]$, and each mode has a zero-point energy $\frac{1}{2}\hbar\omega$, there is an electromagnetic energy density

$$(\omega^2/\pi^2c^3)(\tfrac{1}{2}\hbar\omega)Vd\omega = (\hbar\omega^3/2\pi^2c^3)Vd\omega \equiv \rho_o(\omega)Vd\omega \qquad (3.1)$$

in the frequency interval $[\omega, \omega + d\omega]$ in the volume V.

Can this vacuum field affect an atom? Consider the rate of stimulated emission in a broadband field of spectral energy density $\rho(\omega)$. This rate for a transition of frequency ω_o is $B\rho(\omega_o)$, where $B = 4\pi^2d^2/3\hbar^2$ is the Einstein B coefficient and d is the transition dipole moment. Thus the <u>stimulated</u> emission rate due to the vacuum field is presumably

$$R_{vf} = B\rho_o(\omega_o) = 2d^2\omega_o^3/3\hbar c^3 = \tfrac{1}{2}A \qquad (3.2)$$

where A is the Einstein A coefficient for <u>spontaneous</u> emission.

In other words, if we are looking for physical manifestations of the vacuum field, it seems that it might have something to do with spontaneous emission - but we only get <u>half</u> the correct rate of spontaneous emission when we calculate the rate of emission stimulated by the vacuum field.

Now let us take a rather different point of view about spontaneous emission. A classical point dipole oscillator feels a radiation reaction field

$$\vec{E}_{RR} = (2e/3c^3)d^3\vec{x}/dt^3 - (\delta m/e)d^2\vec{x}/dt^2 \qquad (3.3)$$

where δm is the electromagnetic mass, and for a dipole oscillator of frequency ω_o this leads to a loss of dipole energy W due to radiation at the rate

$$dW/dt = -(e^2\omega_o^2/3mc^3)W \qquad (3.4)$$

To translate this into quantum theory we must "weight" (3.4) by the oscillator strength $f = 2md^2\omega_o/\hbar e^2$ of the transition. [4] Then we arrive at the emission

rate

$$R_{rr} = fW^{-1}|dW/dt| = (e^2\omega_o^2/3mc^3)(2md^2\omega_o/\hbar e^2) = 2d^2\omega_o^3/3\hbar c^3 = \tfrac{1}{2}A \qquad (3.5)$$

due to radiation reaction. Note that $R_{rr} = R_{vf}$ and that the A coefficient for spontaneous emission is just

$$A = R_{vf} + R_{rr} \qquad (3.6)$$

This suggests that both the vacuum field and radiation reaction are important in spontaneous emission. This inference is correct, as we will see later.

4. The Lamb Shift

An energy $W = -\tfrac{1}{2}\vec{d}\cdot\vec{E}$, where $\vec{d}$ is the induced dipole moment, is associated with a polarizable particle in an electric field $\vec{E}$. Writing $\vec{d} = \alpha(\omega)\vec{E}_\omega$ for the induced dipole moment, where $\alpha(\omega)$ is the polarizability, we have $W = -\tfrac{1}{2}\alpha(\omega)E_\omega^2$ and, if there is a continuous distribution of field frequencies,

$$W = -\tfrac{1}{2}\int\alpha(\omega)[4\pi\rho(\omega)d\omega] \qquad (4.1)$$

where we have taken $E_\omega^2 = 4\pi\rho(\omega)d\omega$.

For an atomic electron in level j, therefore, we expect the level shift

$$W_j = -2\pi\int d\omega\alpha_j(\omega)\rho_o(\omega) \qquad (4.2)$$

due to the vacuum, where $\alpha_j(\omega)$ is the polarizability of level j. In other words W_j is the shift in energy - from the "bare" energy determined by the Schrödinger equation without coupling to radiation - due to the vacuum field.

In other words still, W_j is the quadratic Stark shift due to the vacuum field. Using (3.1), and the Kramers-Heisenberg formula

$$\alpha_j(\omega) = (2/3\hbar) \sum_i d_{ij}^2 \omega_{ij} (\omega_{ij}^2 - \omega^2)^{-1} \tag{4.3}$$

for the polarizability, we arrive at the expression

$$W_j = - (2/3\pi c^3) \sum_i d_{ij}^2 \omega_{ij} \int d\omega \omega^3 (\omega_{ij}^2 - \omega^2)^{-1} \tag{4.4}$$

The limit of a free electron may be obtained by taking the transition frequencies $|\omega_{ij}|$ between electron bound states to be small compared with photon frequencies ω in (4.4):

$$W_{free} = (2/3\pi c^3) \sum_i d_{ij}^2 \omega_{ij} \int d\omega \omega \tag{4.5}$$

The Thomas-Reiche-Kuhn sum rule, $(2m/3\hbar)\sum_i d_{ij}^2 \omega_{ij} = 1$, allows us to write $W_{free} = (e^2\hbar/\pi mc^3)\int d\omega\omega$. This is in fact just the vacuum field expectation value of the term $e^2\vec{A}^2/2mc^2$ in the Hamiltonian. In any case W_{free} is independent of the electron state j, and we therefore regard the difference $W_j - W_{free}$ as the measurable level shift W_j' of an electron in state j:

$$W_j' = W_j - W_{free} = - (2/3\pi c^3) \sum_i d_{ij}^2 \omega_{ij}^3 \int_0^\Omega d\omega \omega (\omega_{ij}^2 - \omega^2)^{-1}$$

$$= (2/3\pi c^3) \sum_i d_{ij}^2 \omega_{ij}^3 \log|\Omega/\omega_{ij}| \tag{4.6}$$

where we have introduced a cutoff frequency Ω in our nonrelativistic approach. (Obviously the nonrelativistic theory will break down at photon frequencies $\omega \approx mc^2/\hbar$.) Taking $\Omega = mc^2/\hbar$, Bethe [5] estimated (4.6) for the hydrogen atom and his estimate was in excellent agreement with the measured Lamb shift. The fact that the "Bethe log" (4.6) works so well implies that the Lamb shift is basically a nonrelativistic, vacuum-field effect. [6]

Once again, however, a different approach is possible (and successful). In deriving the Bethe log we began with the expression $W = -\frac{1}{2}\vec{d}\cdot\vec{E}$ and took $\vec{E}$ to be the vacuum electric field. Now let us forget the vacuum and use the radiation reaction field for $\vec{E}$. For a point dipole $\vec{d}(t)$ this field is easily shown to be given by $\vec{E}_{RR} = \vec{E}_{RR}^{(+)} + \vec{E}_{RR}^{(-)}$, where [7]

$$\vec{E}_{RR}^{(+)} = -(2\pi/V)\sum_{\vec{k},\lambda}\vec{e}_{\vec{k},\lambda}\vec{e}_{\vec{k},\lambda}\cdot\int_0^t dt_1\ddot{\vec{d}}(t_1)e^{i\omega_k(t_1 - t)} \tag{4.7}$$

is the positive-frequency part of $\vec{E}_{RR}$ and $\vec{E}_{RR}^{(-)} = \vec{E}_{RR}^{(+)\dagger}$ is the negative-frequency part. Here the sum is over all field modes for the quantization volume V, and $\vec{e}_{\vec{k},\lambda}$ is a unit polarization vector for a mode with wave vector $\vec{k}$ and polarization index λ (= 1,2). (We can evaluate (4.7) explicitly and arrive at (3.3) with $\vec{d} = e\vec{x}$, but instead we take a perturbative approach that allows us to calculate a quantum level shift.) Then

$$W = -\frac{1}{2}\vec{d}\cdot\vec{E}_{RR} = -\mathrm{Re}[\vec{d}\cdot\vec{E}_{RR}^{(+)}] = 2\pi\mathrm{Re}\sum_{\vec{k},\lambda}e_{\vec{k}\lambda m}e_{\vec{k}\lambda n}\int_0^t dt_1 d_m(t)\dot{d}_n(t_1)e^{i\omega_k(t_1 - t)} \tag{4.8}$$

where m,n (= 1,2,3) label Cartesian coordinates and we follow the summation

convention for repeated indices m,n.

To carry these considerations over to quantum theory we replace $d_m(t)\dot{d}_n(t_1)$ in (4.8) by the expectation value $\langle j|d_m(t)\dot{d}_n(t_1)|j\rangle$. In the approximation of unperturbed motion this is just $i\omega_{ji}d_{jim}d_{ijn}\exp[i\omega_{ji}(t-t_1)]$ summed over a complete set of states i, [7] and so for an atom in state j the level shift due to radiation reaction is

$$W_j = (2\pi/V)\sum_i \sum_{\vec{k},\lambda} \omega_{ji}|\vec{e}_{\vec{k}\lambda}\cdot\vec{d}_{ij}|^2 \int_0^t dt_1 \sin(\omega_k-\omega_{ji})(t_1-t)$$

$$= -(2\pi/V)\sum_i \sum_{\vec{k},\lambda} \frac{\omega_{ji}}{\omega_k-\omega_{ji}}|\vec{e}_{\vec{k}\lambda}\cdot\vec{d}_{ij}|^2 \qquad (4.9)$$

where we have used

$$\int_0^t dt_1 \sin(\omega_k-\omega_{ji})(t_1-t) \cong -P\left(\frac{1}{\omega_k-\omega_{ji}}\right) \qquad (4.10)$$

for $t \gg |\omega_{ji}|^{-1}$, where P denotes the Cauchy principal part implicit in (4.9).

In the mode continuum limit $\sum_{\vec{k},\lambda} \to (V/8\pi^3)\int d^3k \sum_\lambda$ we obtain from (4.9) the expression

$$W_j = -(2/3\pi c^3)\sum_i \omega_{ji}d_{ji}^2 \int_0^\Omega \frac{d\omega\omega^2}{\omega-\omega_{ji}} \qquad (4.11)$$

and therefore

$$W'_j{}' = W'_j - W_{free} = -(2/3\pi c^2)\sum_i \omega_{ji}^2 d_{ji}^2 \int_0^\Omega \frac{d\omega\omega}{\omega - \omega_{ji}} \qquad (4.12)$$

This is not the Bethe log expression (4.6); one more subtraction, namely mass renormalization, is required. This is perhaps not surprising, since we are dealing now with the radiation reaction field, which brings in an "extra" electromagnetic mass. The electromagnetic mass term in (3.3) implies that we should subtract from (4.12) the energy

$$\Delta W_j = \langle j|p^2/2m - p^2/2m_o|j\rangle \cong -(\delta m/m)\langle j|p^2/2m|j\rangle = -(2e^2\Omega/3m^2c^3)\langle j|p^2|j\rangle \qquad (4.13)$$

in order to avoid "double counting" the electromagnetic self-energy in our radiation reaction approach. Here m and m_o are respectively the renormalized and bare mass of the electron ($m = m_o + \delta m$). Thus

$$W'_j = W'_j{}' - \Delta W_j = (2/3\pi c^3)\sum_i d_{ij}^2 \omega_{ij}^3 \log|\Omega/\omega_{ij}| \qquad (4.14)$$

which is precisely the Bethe expression (4.6).

What this hodgepodge of classical and quantum considerations shows is that the main portion of the Lamb shift may also be attributed to the radiation reaction field of the atomic electron. It should be clear by now what I am leading up to: <u>effects usually attributed to vacuum field fluctuations may instead be attributed to radiation reaction</u>.

5. <u>Casimir Effects</u>

When we consider Casimir effects this alternative explanation seems at

first implausible. Such effects are typically explained in terms of changes in the vacuum field configuration due to the presence of matter. These changes result in forces between material systems such as polarizable particles or conducting plates.

The thing that makes a source-field interpretation seem implausible at first is that these forces depend on distances between particles and/or plates, and we are used to thinking about radiation reaction in idealized free space, where of course $\vec{E}_{RR}$ does not depend on where a particle is located. But in general radiation does depend on where the particle is located.

Consider, for instance, an atom at a distance d from a perfectly conducting plate. For the z-component of the radiation reaction field we obtain [7]

$$E_{RR,z} = (2e/3c^3)d^3z/dt^3 - (4\Omega/3\pi c^3)d^2z/dt^2 - (2e/4d^2c)\dot{z}(t-2d/c) - (2e/8d^3)z(t-2d/c) \qquad (5.1)$$

This is nothing but the free-space ($d \to \infty$) result plus the retarded dipole field from an image atom at a distance d behind the plate. The positive-frequency part of the source field (5.1) is

$$E^{(+)}_{RR,z} = -(2e/\pi c^3)\int d\omega\omega^2(1/3 - \cos\omega x/\omega^2x^2 + \sin\omega x/\omega^3x^3) \times \int_0^t dt'\dot{z}(t')e^{i\omega(t'-t)} \qquad (5.2)$$

where $x \equiv 2d/c$. Therefore from (4.8) the distance-dependent portion of W for an atom in state j is

$$W_j(d) = -(2e^2/\pi c^3)\int d\omega\omega^2(\cos\omega x/\omega^2x^2 - \sin\omega x/\omega^3x^3) \times \int_0^t dt' \langle j|z(t)\dot{z}(t')|j\rangle e^{i\omega(t'-t)} + \ldots \qquad (5.3)$$

where ... indicates contributions from the x and y components of the atomic dipole. In the approximation of unperturbed motion discussed just before equation (4.9) we obtain

$$W_j(d) = e^2/2\pi i \sum_i \omega_{ji}|z_{ji}|^2 \int_0^\infty dk k^2 (kc - \omega_{ji})^{-1} \frac{e^{2ikd}}{2kd}(2i/2kd - 2/4k^2d^2) + c.c. + \ldots \qquad (5.4)$$

This is the expression obtained by Casimir and Polder in 1948. [8] They evaluated (5.4) in a standard way by introducing a factor $e^{-\gamma k}$ and then taking the limit $\gamma \to 0^+$ after doing the integral. This led to the Casimir-Polder energy

$$W_j(d) = -3\alpha_j\hbar c/8\pi d^4 \qquad (5.5)$$

where α_j is the static ($\omega \to 0$) polarizability for state j. Thus, by allowing for the position dependence of radiation reaction (equation (5.1)), we can derive the Casimir-Polder force.

What about the Casimir force between two conducting plates? The approach based on vacuum field fluctuations considers the total field energy ($\frac{1}{2}\hbar\omega$ per mode) when the plates are separated by a distance d, minus the energy when $d \to \infty$, and this leads to the famous Casimir force [9]

$$F = -\pi\hbar c/480d^4 \qquad (5.6)$$

per unit area. This is a macroscopic approach in that it deals with field modes satisfying boundary conditions, without treating the plates in any sort of "atomistic" fashion. That is, the plates serve only to define the boundary conditions. In a similar fashion one can carry out a macroscopic source field approach, making no reference at all to the vacuum field. This has been worked out by Schwinger, et al. within the context of Schwinger's source theory, "where the vacuum is regarded as truly a state with all physical properties equal to zero." [10] The point, again, is that we can explain a "vacuum field effect" in terms of source fields rather than vacuum fields.

6. Van der Waals Forces

In general the (Coulomb-gauge) vector potential operator may be written in the form

$$\hat{\vec{A}}(\vec{x},t) = \sum_{\alpha} (2\pi\hbar c^2/\omega_\alpha)^{1/2}[\hat{a}_\alpha(t)\vec{F}_\alpha(\vec{x}) + \hat{a}_\alpha^\dagger(t)\vec{F}_\alpha^*(\vec{x})] \tag{6.1}$$

where the (orthonormal) mode functions $\vec{F}_\alpha(\vec{x})$ satisfy the Helmholtz equation

$$\nabla^2\vec{F}_\alpha(\vec{x}) + k_\alpha^2\vec{F}_\alpha(\vec{x}) = 0,\ k_\alpha^2 = \omega_\alpha^2/c^2 \quad , \tag{6.2}$$

the transversality condition $\nabla\cdot\vec{F}_\alpha(\vec{x}) = 0$, and the appropriate boundary conditions for the electric and magnetic fields; $\hat{a}_\alpha$ and $\hat{a}_\alpha^\dagger$ are the annihilation and creation operators for mode α, and the caret (^) is used to indicate an operator.

In the presence of sources the Heisenberg equation of motion for $\hat{a}_\alpha(t)$ leads to the following expression for the vector potential corresponding to

the radiation reaction field acting on a point particle of charge e:

$$\vec{A}_{RR}(\vec{x},t) = 2\pi iec \sum_{\alpha} \omega_{\alpha}^{-1}\vec{F}_{\alpha}(\vec{x})\cdot\vec{F}_{\alpha}^{*}(\vec{x}) \int_{0}^{t} dt'\,\hat{\vec{r}}(t')e^{i\omega_{\alpha}(t'-t)} + h.c. \qquad (6.3)$$

In the case of free space we have $\vec{F}_{\alpha}(\vec{x}) = V^{-1/2}\vec{e}_{k\lambda}e^{i\vec{k}\cdot\vec{x}}$, and we can evaluate (6.3) and use $\vec{E}_{RR} = -(1/c)\vec{A}_{RR}$ to obtain (3.3).

In the preceding section the appropriate mode functions $\vec{F}_{\alpha}(\vec{x})$ were those appropriate to a half-space bounded by a perfectly conducting plane. Using these modes, we obtain (5.1); of course we can also write (5.1) on physical grounds, without resorting to a modal decomposition of the field.

Now for the van der Waals interaction between two atoms we are interested in the radiation reaction field on a ground-state atom A when there is some other atom B a distance d away. So we want to know how atom B affects the modes of the field acting on A. A ground-state atom B modifies the free-space mode functions by adding to each plane wave a dipole field due to the scatterer:

$$\vec{e}_{k\lambda}e^{i\vec{k}\cdot\vec{x}} \to \vec{e}_{k\lambda}e^{i\vec{k}\cdot\vec{x}} + \alpha_B(\omega_k)k^3 e^{i\vec{k}\cdot\vec{x}_B}\, e^{ikr}[\vec{e}_{k\lambda}(1/kr + 1/k^2r^2 - 1/k^3r^3)$$

$$- r^{-2}(\vec{e}_{k\lambda}\cdot\vec{r})\vec{r}(1/kr + 3i/k^2r^2 - 3/k^3r^3)] \qquad (6.4)$$

where $\vec{x}_B$ specifies the position of atom B, $\vec{r} = \vec{x} - \vec{x}_B$, and $\alpha_B(\omega)$ is the polarizability of the ground state of atom B. Using these modified (by atom B) mode functions, we can calculate the radiation reaction field for atom A in the presence of atom B, and therefore the radiative level shifts of atom A. For the ground-state level shift of atom A we obtain just the London result

W(d) proportional to d^{-6} for $d \to 0$, whereas for large d we obtain the retarded van der Waals interaction proportional to d^{-7} [8]:

$$W(d) = - 23\hbar c \alpha_A \alpha_B / 4\pi d^7 \qquad (6.5)$$

where α_A and α_B are the static polarizabilities of the atoms. Thus the van der Waals interaction may be understood from the perspective of radiation reaction, provided we recognize that the radiation reaction field for atom A depends on the position and polarizability of atom B. That is, we must keep in mind, as in the preceding section, that the field of radiation reaction acting on a particle depends on the modal properties of the field, and consequently on the electromagnetic environment of the particle.

7. Cavity-Modified Spontaneous Emission

If the radiation reaction field depends on the environment of the emitter, then so too will the radiation rate. This means that an excited atom near a conducting wall or inside a cavity, for example, will have a spontaneous emission rate different from the Einstein A coefficient appropriate to free space.

This cavity-modified spontaneous emission has been observed experimentally. Delightfully simple experiments by Drexhage, [11] for instance, involved the deposition of molecular monolayers on reflecting plates. The distance of a fluorescing molecule from the plate could be set fairly accurately according to the number of times the plate was dipped in a solution before the monolayer was added. In this way the emission rate could be monitored as a function of the distance of the emitter from the reflecting plate. The theory is quite simple if idealized conditions are assumed, and in fact the alteration of the emission rate could be predicted using a simple,

classical picture of a dipole oscillator near a reflecting wall.

More recent experimental work in this area of environmentally modified spontaneous emission has been undertaken by Kleppner, [12] Haroche, [13] and DeMartini [14] and their collaborators.

The field (6.1) is the total field. That is, $a_\alpha(t)$ has a homogeneous solution plus a source term. The source term is associated with radiation reaction, whereas the homogeneous part represents the external field. The homogenous solution must always be accounted for, because even in the absence of an applied field there is still the vacuum field. The mode expansion (6.1) then shows that the vacuum field, like the radiation reaction field, depends on the mode functions $\vec{F}_\alpha(\vec{x})$, and therefore on the atom's environment. If we choose to think about spontaneous emission in terms of vacuum field fluctuations, then that environmental dependence explains cavity-modified spontaneous emission.

This seems so obvious, and yet when Drexhage first reported his experimental results some theorists argued that no modification of a spontaneous emission rate was possible: how could the spontaneous emission of a photon be affected by the atom's environment, since the atom can only "see" its surroundings by emitting a photon in the first place? The gainsayers did not understand very deeply what a photon is. I also recall a sentence from my undergraduate modern physics textbook: "The transition rate for spontaneous emission is an inherent characteristic of the atom and is not influenced by the environment in which the atom is placed." [15] That sentence was deleted in subsequent editions.

8. Two Sides of a Coin

How is it that we can explain these effects in terms of radiation reaction, when for nearly half a century people have successfully explained

them in terms of vacuum field fluctuations.?

The answer to this question was found independently and practically simultaneously in 1973 by Senitzky, [16] Smith, [17] and the present author. [17,18] The motivation for this work grew out of E.T. Jaynes' neoclassical theory of spontaneous emission. [19] In this theory spontaneous emission was attributed to radiation reaction, but the radiation reaction field was treated as a classical field, and it was assumed that there is no vacuum (source-free) field. (This is really an assumption, not an inevitable feature of classical field theory.) Although the neoclassical theory explains some important features of spontaneous emission, it was deficient in certain respects; for instance, it could not account for the observed photon polarization correlations in a three-level cascade. [20] In 1972 Ackerhalt, et al. [21] showed that radiation reaction nevertheless offers a valid basis for understanding spontaneous emission, provided the radiation reaction field is handled properly as a quantum-mechanical operator. This showed that radiation radiation was a legitimate way of understanding spontaneous emission, and the question then was to reconcile such an interpretation with the old idea that spontaneous emission is triggered by vacuum field fluctuations.

It was shown in the case of spontaneous emission [16,17] that the physical interpretation suggested by quantum electrodynamics is more or less a consequence of the way we choose to order commuting atomic and field operators. For instance, a normal ordering emphasizes the role of radiation reaction, because the vacuum field does not contribute to the expectation values of interest; this was the case in the work of Ackerhalt, et al. However, the role of the vacuum field could be emphasized by using a symmetrical ordering of atomic and field operators, for then expectation values of vacuum-field operators (e.g., $\langle aa^{\dagger} \rangle$) are found to make explicit contributions to the radiative decay and level shifts.

It would be inappropriate to reproduce here the algebra leading up to this reconciliation of the vacuum and source-field interpretations, since it is readily available in the literature. [16-18, 22] I will just list a few salient points:

(1) The level shifts and widths can be attributed exclusively to radiation reaction or the vacuum field, or to linear combinations of the two. For instance, we can say that the major (nonrelativistic) portion of the Lamb shift is i parts a source-field effect and (1 - i) parts a vacuum field effect.

(2) There is no ordering that attributes the radiative decay of a level entirely to the vacuum field.

(3) All these interpretational results rest on the usual sort of weak-coupling approximations used in the quantum theory of decaying states.

(4) These results are obtained in the Heisenberg picture, where questions of physical interpretion are most conveniently addressed by analcgy with classical theory. In the Schrödinger picture, on the other hand, one can calculate the same things but the physical interpretation is less amenable to classical-like interpretations.

(5) The vacuum field and radiation reaction may be regarded in a sense as "two sides of the same coin." [16] There is an underlying reason for this intimate connection, namely a fluctuation-dissipation relation that can be expressed in terms of commutation relations.

We now turn our attention to the last point.

9. Commutators

The intimate connection between radiation reaction and the vacuum field may be traced to the fact that the vacuum spectral energy density $\rho_0(\omega)$ goes as the third power of ω (equation (3.1)), while the radiation reaction field goes as the third derivative of $\vec{x}$ after mass renormalization (equation (3.3)). This in turn is due to the fact that both fields derive from the same mode expansion (6.1). (In principle, of course, a mode expansion is unnecessary, but that's beside the point for our discussion here.)

Indeed this kinship is required for the very consistency of the quantum theory of radiation. Consider the Heisenberg equation of motion for a nonrelativistic electron in free space:

$$d^2\hat{x}/dt^2 - \tau d^3\hat{x}/dt^3 = (e/m)\hat{E}_0 \tag{9.1}$$

where $\tau = 2e^2/3mc^3$ and $\hat{E}_0$ is the "vacuum" electric field operator, i.e., the homogeneous solution of the Maxwell (Heisenberg) equation for the electric field strength. Writing $\hat{E}_0(t)$ as a sum over all field modes, and then solving for $\hat{x}(t)$, it is easily shown that [7]

$$[\hat{x}(t),\hat{p}(t)] = (8\pi^2 i/3m)\int_0^\infty d\omega \rho_0(\omega)/[\omega^3(1+\tau^2\omega^2)] = i\hbar \tag{9.2}$$

This result is no surprise, but it puts into sharper focus the kinship of the vacuum and source fields. If $\hat{E}_{RR}$ varied as $d\hat{x}/dt$ instead of $d^3\hat{x}/dt^3$, for instance, then $\rho_0(\omega)$ would have to be proportional to ω instead of ω^3 to maintain the canonical commutation relation between $\hat{x}$ and $\hat{p}$.

In fact this may be understood from the general fluctuation-dissipation theorem [20] for linearly dissipative systems. [7, 21] Indeed we can derive the (T = 0) fluctuation-dissipation relation in general by demanding the preservation of canonical commutation rules in the presence of a dissipative (fluctuating) force. Whichever way we look at it, the intimate relation between $\hat{E}_o$ and $\hat{E}_{RR}$ is required for the logical consistency of the quantum theory of matter-field interactions. The work reported in References [16 - 18] was in a sense a rediscovery, in a specific context, of the fluctuation-dissipation connection. Incidentally this connection makes it clear that if the vacuum field depends on the electromagnetic environment, then so too must the radiation reaction if commutators are to be preserved everywhere.

(Recently it has been written that some "false assumptions" have been made in the evaluation of $[\hat{x}, \hat{p}]$ from the equation of motion of an electron in the field $\hat{E}_o + \hat{E}_{RR}$. [23] The objection appears to center on my ignoring the homogeneous solution, $\hat{x}_{hom}$, of (9.1) in calculating (9.2). Being aware of the fact that the solution of a differential equation is composed of a homogeneous part and an inhomogeneous part, I wish to note here that the important assumption is the independence of $\hat{x}_{hom}$ and $\hat{E}_o$. Under this assumption $[\hat{x}_{hom}, \hat{E}_o]$ is identically zero and it is easy to show then that $[\hat{x}(t), \hat{p}(t)]$ is equal to (9.2). In any case the consideration of $\hat{x}_{hom}$ in no way affects the fluctuation-dissipation connection.)

10. Accelerated Observers

An atom in a thermal field characterized by a temperature T has its radiative level widths and shifts modified from their zero-temperature values. For instance, the spontaneous emission rate for a transition of frequency ω_o becomes

$$A' = A(2\bar{n} + 1) \tag{10.1a}$$

$$\bar{n} = (e^{\hbar\omega_o/kT} - 1)^{-1} \tag{10.1b}$$

where A is the usual (T = 0) emission rate. This temperature effect is negligible for practical purposes because $\bar{n}$ is very small at optical frequencies.

Now when a physical system is uniformly accelerated in vacuum it acts as if it were immersed in a ther th at temperature [24]

$$T_a = \hbar a/2\pi kc \tag{10.2}$$

where a is the (constant) acceleration re'ative to an inertial frame and k is Boltzmann's constant. This remarkable effect has been examined in detail by Sciama, et al., [25] and has been elucidated from the standpoint of random electrodynamics by Boyer. [26] Based on this result we might expect that for a uniformly accelerated atom the spontaneous emission rate becomes

$$A' = A(2n_a + 1) \tag{10.3a}$$

$$n_a = (e^{\hbar\omega_o/kT_a} - 1)^{-1} \tag{10.3b}$$

Let us now see what we get by doing a calculation.

Consider a two-state atom coupled to the electromagnetic field through the interaction $-e\vec{r}\cdot\vec{E}$. The Heisenberg equations of motion for the two-state transition operators σ and σ_z are [27]

$$\dot{\hat{\sigma}}(t) = -i\omega_o\hat{\sigma}(t) - (i/\hbar)\mu_j[\hat{\sigma}_z(t)\hat{E}_j^{(+)}(t) + \hat{E}_j^{(-)}(t)\hat{\sigma}_z(t)] \tag{10.4}$$

$$\dot{\hat{\sigma}}_z(t) = -(2i/\hbar)\mu_j[\hat{E}_j^{(-)}(t)\hat{\sigma}(t) - \hat{\sigma}^\dagger(t)\hat{E}_j^{(+)}(t)] \tag{10.5}$$

where $\vec{\mu}$ is the transition dipole moment and $\hat{E}^{(+)}(t)$ and $\hat{E}^{(-)}(t)$ are respectively the positive- and negative-frequency parts of the total (free field plus source) electric field operator. In writing (10.4) and (10.5) we have normally ordered these operators and made the "rotating-wave approximation" of neglecting terms such as $\hat{\sigma}\hat{E}^{(+)}$, which corresponds to the simultaneous lowering of both atomic and field excitation. We have the following expression for $\hat{E}_i^{(+)}(t)$, with $\hat{E}_i^{(-)} = \hat{E}_i^{(+)}(t)^\dagger$:

$$\hat{E}_i^{(+)}(t) = \hat{E}_{oi}^{(+)}(t) + \hat{E}_{si}^{(+)}(t) \tag{10.6}$$

where $\hat{E}_o$ is the vacuum electric field operator and $\hat{E}_s$ is the field due to the source (atom):

$$\hat{E}_{sj}^{(+)}(t) = (i/4\pi^2)\int d^3k\omega\mu_i(\delta_{ij} - k_ik_j/k^2)\int_0^t dt'[\hat{\sigma}(t') + \hat{\sigma}^\dagger(t')]e^{i\omega(t'-t)} \tag{10.7}$$

For present purposes it is convenient to cast (10.7) in a somewhat different form:

$$\hat{E}_{sj}^{(+)}(t) = (i/\hbar)\mu_i\int_0^t dt'\hat{\sigma}_x(t')\langle\hat{E}_{oi}^{(+)}(t)\hat{E}_{oj}^{(-)}(t')\rangle \tag{10.8}$$

where $\hat{\sigma}_x \equiv \hat{\sigma} + \hat{\sigma}^\dagger$ and the expectation value on the right refers to the vacuum

state of the field. (The atomic state is arbitrary because $\hat{E}_{oj}^{(\pm)}(t)$ does not act on atomic states.) Since

$$\hat{E}_{oj}^{(+)}(t)|vac\rangle = \langle vac|\hat{E}_{oj}^{(-)}(t) = 0 \tag{10.9}$$

we can write (10.8) equivalently as

$$\hat{E}_{sj}^{(+)}(t) = (i/\hbar)\mu_i \int_0^t dt' \hat{\sigma}_x(t')\langle \hat{E}_{oi}(t)\hat{E}_{oj}(t')\rangle \tag{10.10}$$

Of course $\hat{E}_o(t)$ is actually the source-free electric field operator $\hat{E}_o(\vec{x},t)$ at the coordinate $\vec{x} = 0$ of the atom, which is taken as a point object in the usual electric-dipole approximation we are employing. Thus

$$\hat{E}_{sj}^{(+)}(t) = (i/\hbar)\mu_i \int_0^t dt' \hat{\sigma}_x(t')\langle \hat{E}_{oi}(0,t)\hat{E}_{oj}(0,t')\rangle \tag{10.11}$$

This equation relates the source field to the free-field correlation function - another example of the intimate relation between radiation reaction and the vacuum field!

Equations (10.4) - (10.6) and (10.11) have no known exact solution. In one form or other virtually all treatments use some variant of the original Weisskopf-Wigner approximation. In the present formulation this amounts to the replacement

$$\hat{\sigma}(t') \to \hat{\sigma}(t)e^{-i\omega_o(t'-t)} \tag{10.12a}$$

$$\hat{\sigma}^\dagger(t') \to \hat{\sigma}^\dagger(t)e^{i\omega_o(t'-t)} \tag{10.12b}$$

in the integral over t' in (10.7); this approximation derives from (10.4) under the assumption of weak (compared with $\hbar\omega_o$) atom-field coupling. Using (10.12) in (10.11), we have

$$\hat{E}^{(+)}_{sj}(t) \cong (i/\hbar)\mu_i \hat{\sigma}(t) \int_0^t dt' \langle \hat{E}_{oi}(0,t)\hat{E}_{oj}(0,t')\rangle e^{-i\omega_o(t'-t)} \tag{10.13}$$

in the rotating-wave approximation, and from (10.4) and (10.9) it then follows that

$$\langle \dot{\hat{\sigma}}(t)\rangle \cong - i\omega_o \langle \hat{\sigma}(t)\rangle - (\mu_i\mu_j/\hbar^2)\langle \hat{\sigma}(t)\rangle \int_0^t dt' \langle \hat{E}_{oi}(0,t)\hat{E}_{oj}(0,t')\rangle e^{-i\omega_o(t'-t)} \tag{10.14}$$

From this we may identify, within the Weisskopf-Wigner approximation, the spontaneous decay rate

$$A = \int_0^t dt' D(t'-t)\cos\omega_o(t'-t) \qquad (t \to \infty) \tag{10.15}$$

where

$$D(t'-t) \equiv (\Sigma\mu_i\mu_j/\hbar^2)\langle \hat{E}_{oi}(0,t)\hat{E}(0,t')\rangle \tag{10.16}$$

For our purposes it will simplify things somewhat to use the correlation function of the vector potential operator and write

$$D_A(t'-t) = (2\mu_i\mu_j/\hbar^2c^2)\langle\hat{A}_{oi}(0,t)\hat{A}_{oj}(0,t')\rangle \tag{10.17}$$

instead of (10.16) in (10.15). (This leads to a simplification of the algebra below.)

Now

$$D_A(t'-t) = G(\vec{x}',t';\vec{x},t)_{\vec{x}'=\vec{x}} \tag{10.18}$$

where

$$G(\vec{x}',t';\vec{x},t) = (\mu^2\omega_o^2/3\pi\hbar c^2)\mathrm{Re}\left[\int_0^\infty d\omega e^{i\omega(t'-t)}\,\frac{\sin\omega|\vec{x}'-\vec{x}|/c}{|\vec{x}'-\vec{x}|}\right]$$

$$= -(\mu^2\omega_o^2/3\pi\hbar c^2)\,[(t'-t)^2 - |\vec{x}'-\vec{x}|/c^2]^{-1} \tag{10.19}$$

I have replaced a sum over polarizations by an effective value for simplicity (and without intending this to be a fully rigorous or complete calculation).

Consider now an atom undergoing uniform acceleration relative to an inertial frame in which it is instantaneously at rest. Its acceleration in the (inertial) lab frame is given by

$$dv/dt = a(1 - v^2/c^2)^{3/2} \tag{10.20}$$

For constant acceleration a, the velocity v and position x in the lab frame follow from (10.20) by simple integrations. We can also obtain the time t in the lab frame from the relation $d\tau = dt\sqrt{1 - v^2/c^2}$ for the proper time interval $d\tau$. The result of these manipulations is the parametrization

$$t(\tau) = (c/a)\sinh(a\tau/c) \tag{10.21a}$$

$$x(\tau) = (c^2/a)\cosh(a\tau/c) \tag{10.21b}$$

if we choose $t(\tau = 0) = 0$ and $x(\tau = 0) = c^2/a$ for this so-called hyperbolic motion. Equations (10.21) imply

$$[(t'-t)^2 - |\vec{x}'-\vec{x}|^2]^{-1} = (a^2/c^2)\mathrm{csch}^2[a(\tau'-\tau)/2c] \tag{10.22}$$

Now for an atom in hyperbolic motion we write (10.15) as

$$A' = \int_0^\infty d\tau' D_A(\tau'-\tau)\cos\omega_o(\tau'-\tau) \tag{10.23}$$

and use (10.18), (10.19), and (10.22):

$$A' = -(\mu^2\omega_o^2a^2/3\pi\hbar c^5)\int_0^\infty d\tau'\,\mathrm{csch}^2[a(\tau'-\tau)/2c]\cos\omega_o(\tau'-\tau) \qquad (\tau \to \infty)$$

$$= (4\mu^2\omega_o^3/3\hbar c^3)\coth(\pi c\omega_o/a) = A\coth(\pi c\omega_o/a) = A\coth(\hbar\omega_o/2kT_a)$$

$$= A(2n_a + 1) \tag{10.24}$$

which is precisely (10.3). That is, <u>the spontaneous emission rate of a uniformly accelerated atom is as if the atom were in a thermal field of temperature $T_a = \hbar a/2\pi kc$</u>.

If we think of spontaneous emission as a consequence of vacuum field fluctuations, we can interpret this result as follows: for an accelerated

observer the quantum vacuum fluctuations are "promoted" to the level of "real" thermal fluctautions. [25]

When there is no acceleration the atom does not absorb energy from the vacuum because the vacuum field fluctuations are effectively cancelled by radiation reaction. [22] For an accelerated atom this balance is broken and there is absorption from the vacuum - to the extent that the atom reaches a Boltzmann distribution at temperature T_a.

11. Where Do We Stand?

Welton in 1948 stated that spontaneous emission "can be thought of as forced emission taking place under the action of the fluctuating [vacuum] field." [6] Over and over again one finds similar remarks in the literature. [28] Yet if we take this idea seriously and calculate the emission rate due to the vacuum field in a naive sort of way, we find only half the Einstein A coefficient. (Section 3) Furthermore this picture offers no explanation as to why there is no spontaneous absorption from the vacuum field. [22]

An older and more classically motivated interpretation of spontaneous emission attributes this phenomenon to radiation reaction. [29] The idea, more or less, is that spontaneous emission is simply a consequence of the fact that oscillating dipoles radiate. But we cannot use classical electrodynamics here - we get erroneous results if we do. [22] For this reason, perhaps, the vacuum-field interpretation eventually won out.

The idea that spontaneous emission may be attributed to the vacuum electromagnetic field has also been criticized by Ginzburg - beginning in 1939 [30] and then again in 1983. [31] In particular, he refers to the "1/2 discrepancy" noted earlier. He also notes, as the present author has on several occasions, [22] that an unobjectionable explanation of spontaneous emission was given by Fermi in his important review article in 1932. [32]

Quoting Ginzburg, "Spontaneous radiation appears because the state in which a mechanical subsystem (an atom, a moving charge, etc.) is at some level ... but the radiation field ... is absent, is not a stationary eigenstate of the complete system (the mechanical subsystem + the electromagnetic field)." (In connection with his 1939 articles on the nature of spontaneous emission, Ginzburg writes that, "These articles were the first ones I ever wrote, and naturally the memory of them as a first love in theoretical physics stimulated me to a significant degree after four decades to write the present note." [31])

I believe it is fair to say that since 1973 we have had a more sophisticated understanding of why spontaneous emission occurs. We now understand in what sense the two older physical interpretations of spontaneous emission were valid, and we can extend the newer interpretation to other "vacuum-field effects," such as van der Waals forces.

From this and other problems in quantum optics we have come to better appreciate the important differences between positive- and negative frequency parts of the field in classical and quantum electrodynamics. If we always use a symmetric ordering of these operators we can see clearly, for instance, why the classical theory of random electrodynamics [33] has enjoyed considerable success. [22] Various other orderings give different weights to the vacuum and source fields when we try to interpret the results of a calculation. To emphasize as much as possible the classical-like aspects of the vacuum and source fields, we can choose a symmetric ordering at every stage of a calculation. [34]

From another point of view all of this is unimportant because we can calculate whatever we need in the Schrödinger or interaction picture, where we are relatively safe from questions of classical-like, intuitive interpretions of spontaneous emission, van der Waals forces, etc. Since I respect that

philosophy but do not myself subscribe to it. I will not discuss it further here.

On the experimental side, it is not possible to distinguish between "vacuum" and "source" effects in this context. What is important, of course, is that these effects are real. However, I should point out that the experimental data for the Casimir force between two plates may not be as conclusive as some of us had thought. Zajonc [35] has noted that the data from Sparnaay's experiments [36] are in better agreement with the London-van der Waals force (d^{-3}) than the Casimir (d^{-4}). He plans to perform such experiments with greater precision.

Is there anything more to be learned, really, about the vacuum electromagnetic field? Obviously it would be presumptuous of me to offer an answer to that question. I will instead suggest that these ideas may provide useful intuitive guides in QCD, where Casimir-type effects have recently been of some interest in connection with quark confinement.

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